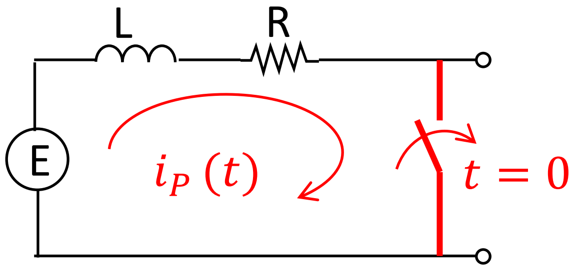
Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

Short Circuit at primary side of CT

$$I_m = \frac{E_m}{Z}$$

$$E_m = \frac{\sqrt{2} \times V_{LL}}{\sqrt{3}} \qquad E = \frac{V_{LL}}{\sqrt{3}} \qquad E(t) = E_m \sin(\omega t + \theta)$$



$$\tan\varphi = \frac{\omega L}{R} = \frac{X}{R}$$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

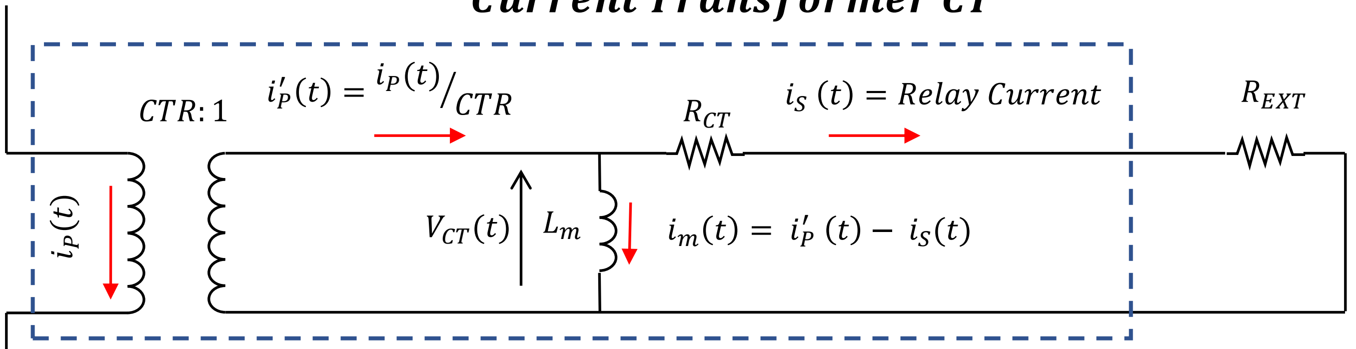
$$T_P = \frac{L}{R}\ (sec - primary\ time\ constant\ of\ SC)$$

Short Circuit Current at CT Primary

$$\boxed{i_P(t) = -I_m \sin(\theta - \varphi)\ e^{-\frac{R}{L}t} + I_m \sin(\omega t + \theta - \varphi)\ ,\ i_P\,(t = 0) = 0}$$

$$\boxed{i_P(t) = -I_m \sin(\theta - \varphi)\ e^{-\frac{t}{T_P}} + I_m \sin(\omega t + \theta - \varphi)\ ,\ i_P\,(t = 0) = 0}$$

Current Transformer CT



$CTR: CT\ Ratio$

$R_{CT} = CT\ internal\ resiatance$

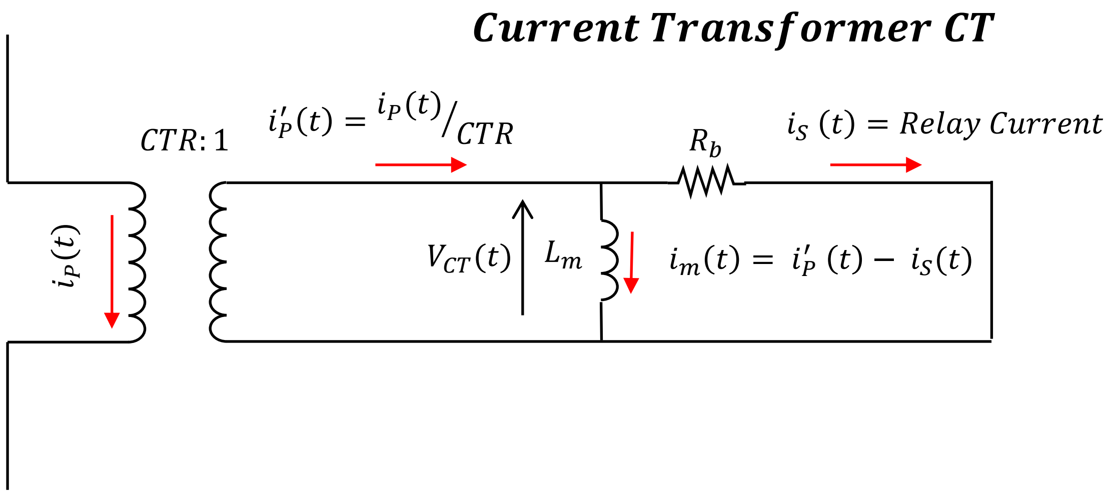
$R_{EXT} = External\ Resistances\ (Cable\ plus\ relay)$

$R_b = R_{CT} + R_{EXT}\ \ \ldots\ldots CT\ total\ burden$

$T_S = secondary\ time\ constant$

$$V_{CT}(t) = i_S\,(t) \times R_b = L_m \frac{di_m(t)}{dt} = L_m \left[\frac{di'_P(t)}{dt} - \frac{di_S(t)}{dt}\right] \rightarrow L_m \frac{di_S(t)}{dt} + R_b\ i_S\,(t) = L_m \frac{di'_P(t)}{dt} \xRightarrow{T_S = \frac{L_m}{R_b}} \boxed{\frac{di_S(t)}{dt} + \frac{1}{T_S}\ i_S\,(t) = \frac{di'_P(t)}{dt}}$$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor



$$i_P(t) = -I_m \sin(\theta - \varphi)\ e^{-\frac{t}{T_P}} + I_m \sin(\omega t + \theta - \varphi)$$

&

$$i'_P(t) = \frac{i_P(t)}{CTR}$$

$$i'_P(t) = -\frac{I_m}{CTR} \sin(\theta - \varphi)\ e^{-\frac{t}{T_P}} + \frac{I_m}{CTR} \sin(\omega t + \theta - \varphi)$$

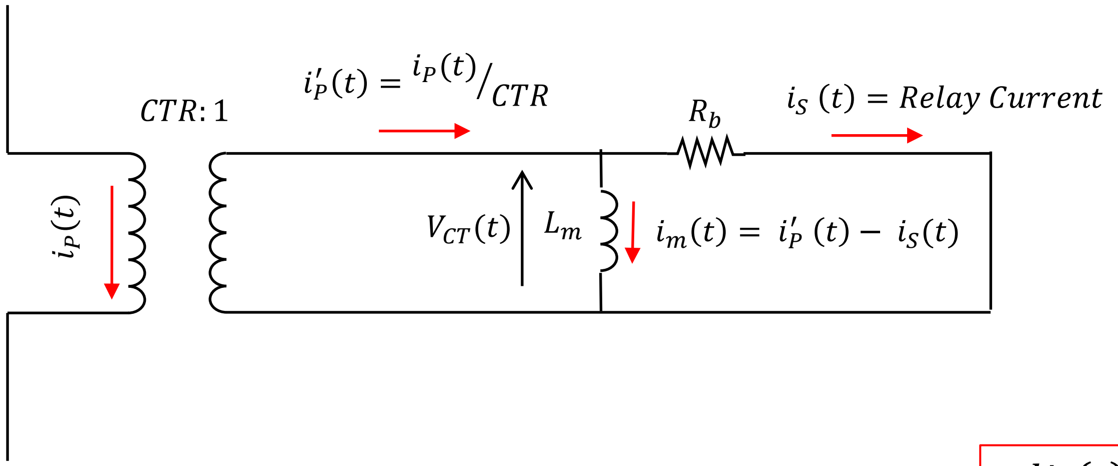
&

$$I'_m = \frac{I_m}{CTR}$$

$$\boxed{i'_P(t) = -I'_m \sin(\theta - \varphi)\ e^{-\frac{t}{T_P}} + I'_m \sin(\omega t + \theta - \varphi)}$$

$$\boxed{\frac{d\, i'_P(t)}{dt} = \frac{I'_m}{T_P} \sin(\theta - \varphi)\ e^{-\frac{t}{T_P}} + I'_m\, \omega\ \cos(\omega t + \theta - \varphi)}$$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor



$$\frac{di_S(t)}{dt} + \frac{1}{T_S}\,i_S\,(t) = \frac{di'_P(t)}{dt}$$

&

$$\frac{d\,i'_P(t)}{dt} = -\frac{I'_m}{T_P}\sin(\theta - \varphi)\;e^{-\frac{t}{T_P}} + I'_m\,\omega\;\cos(\omega t + \theta - \varphi)$$

$$\boxed{\frac{di_S(t)}{dt} + \frac{1}{T_S}\,i_S\,(t) = -\frac{I'_m}{T_P}\sin(\theta - \varphi)\;e^{-\frac{t}{T_P}} + I'_m\,\omega\;\cos(\omega t + \theta - \varphi)}$$

Refer Appendix A – Details of calculation of $i_S(t)$

$$\boxed{i_S(t) = \frac{T_P}{T_S - T_P}I'_m\sin(\theta - \varphi)\,e^{-\frac{t}{T_S}} - \frac{T_S}{T_S - T_P}\,I'_m\sin(\theta - \varphi)\,e^{-\frac{t}{T_P}} + I'_m\;\sin(\omega t + \theta - \varphi)}$$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

$\boldsymbol{Calculating\ V_{CT}(t)}$

$$\boxed{V_{CT}(t) = R_b \times i_S(t) = R_b \times \left[\frac{T_P}{T_S-T_P} I'_m \sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} - \frac{T_S}{T_S-T_P} I'_m \sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + I'_m \sin(\omega t+\theta-\varphi)\right]}$$

$\boldsymbol{Calculating\ CT\ Flux\ \emptyset_{CT}(t)}$

$$V_{CT}(t) = -\frac{d\,\emptyset_{CT}(t)}{dt} \qquad So:\ \ \emptyset_{CT}(t) = \int -V_{CT}(t)\ dt$$

$$\emptyset_{CT}(t) = \int \left\{R_b \times \left[-\frac{T_P}{T_S-T_P} I'_m \sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} + \frac{T_S}{T_S-T_P} I'_m \sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} - I'_m \sin(\omega t+\theta-\varphi)\right]\right\} dt$$

$$\emptyset_{CT}(t) = R_b \times \left[\frac{T_S\,T_P}{T_S-T_P} I'_m \sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} - \frac{T_S\,T_P}{T_S-T_P} I'_m \sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + \frac{I'_m}{\omega} \cos(\omega t+\theta-\varphi)\right] + Me^{-\frac{t}{T_S}}$$

$Me^{-\frac{t}{T_S}}$ *is a term for initial or remanent flux* $(\emptyset_{CT}\ @t=0)$. *Remanent flux* $\emptyset_R$ *is normaly shown as per unit of*

of knee point flux $\emptyset_K$ *for example* $\emptyset_R = 0.3\,\emptyset_K$

for $\emptyset_R = K_R\,\emptyset_K$ *where* K_R *is a per unit number then*

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

$$K_R \emptyset_K = R_b \times \left[\frac{T_S\, T_P}{T_S - T_P} I'_m \sin(\theta - \varphi) - \frac{T_S\, T_P}{T_S - T_P}\, I'_m \sin(\theta - \varphi) + \frac{I'_m}{\omega} \cos(\theta - \varphi) \right] + M$$

$$K_R \emptyset_K = R_b \times \frac{I'_m}{\omega} \cos(\theta - \varphi) + M$$

$$M = K_R \emptyset_K - R_b \times \frac{I'_m}{\omega} \cos(\theta - \varphi)$$

$$\emptyset_{CT}(t) = R_b \times \frac{I'_m}{\omega} \times \left[\frac{\omega T_S\, T_P}{T_S - T_P} \sin(\theta - \varphi)\, e^{-\frac{t}{T_S}} - \frac{\omega T_S\, T_P}{T_S - T_P} \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}} + \cos(\omega t + \theta - \varphi) \right] + \left[K_R \left(\frac{\emptyset_K}{R_b \times \frac{I'_m}{\omega}} \right) - \cos(\theta - \varphi) \right] e^{-\frac{t}{T_S}}$$

$\emptyset_{AC-Max} = R_b \times \frac{I'_m}{\omega}$ *then* $\emptyset_{CT}(t)$ *equation can be re – write*

$$\emptyset_{CT}(t) = \emptyset_{AC-Max} \times \left\{ \left[\frac{\omega T_S\, T_P}{T_S - T_P} \sin(\theta - \varphi)\, e^{-\frac{t}{T_S}} - \frac{\omega T_S\, T_P}{T_S - T_P} \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}} + \cos(\omega t + \theta - \varphi) \right] + \left[K_R \frac{\emptyset_K}{\emptyset_{AC-Max}} - \cos(\theta - \varphi) \right] e^{-\frac{t}{T_S}} \right\}$$

At knee point this ratio is valid : $K_S = \frac{\emptyset_K}{\emptyset_{AC-Max}} = \frac{V_K}{V_{AC-Max}} = \frac{V_K}{R_b \times I'_m}$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

$$Multiplying\ \frac{\emptyset_K}{\emptyset_K}\ to\ \emptyset_{CT}(t)$$

$$\emptyset_{CT}(t) = \emptyset_{AC-Max} \times \frac{\emptyset_K}{\emptyset_K} \times \left\{\left[\frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} - \frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + \cos(\omega t+\theta-\varphi)\right] + \left[K_R\frac{\emptyset_K}{\emptyset_{AC-Max}} - \cos(\theta-\varphi)\right] e^{-\frac{t}{T_S}}\right\}$$

$$\emptyset_{CT}(t) = \left[\frac{\emptyset_K}{\frac{\emptyset_K}{\emptyset_{AC-Max}}}\right] \times \left\{\left[\frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} - \frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + \cos(\omega t+\theta-\varphi)\right] + \left[K_R\frac{\emptyset_K}{\emptyset_{AC-Max}} - \cos(\theta-\varphi)\right] e^{-\frac{t}{T_S}}\right\}$$

$$\emptyset_{CT}(t) = \frac{\emptyset_K}{K_S} \times \left\{\left[\frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} - \frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + \cos(\omega t+\theta-\varphi)\right] + [K_R K_S - \cos(\theta-\varphi)]e^{-\frac{t}{T_S}}\right\}$$

$\boldsymbol{Time\ to\ saturation\ (t_S)}$

$Time\ to\ saturation\ (t_S) is\ the\ first\ instant\ that\ CT\ flux\ or\ CT\ voltage\ reach\ to\ the\ \emptyset_K\ or\ V_K$:

$$\emptyset_{CT}(t=t_S) = \emptyset_K = \frac{\emptyset_K}{K_S} \times \left\{\left[\frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t_S}{T_S}} - \frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t_S}{T_P}} + \cos(\omega t_S+\theta-\varphi)\right] + [K_R K_S - \cos(\theta-\varphi)]e^{-\frac{t_S}{T_S}}\right\}$$

$or\ at\ t_S$:

$$\boxed{K_S = \frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t_S}{T_S}} - \frac{\omega T_S\, T_P}{T_S - T_P}\sin(\theta-\varphi)\, e^{-\frac{t_S}{T_P}} + \cos(\omega t_S+\theta-\varphi) + [K_R K_S - \cos(\theta-\varphi)]e^{-\frac{t_S}{T_S}}}$$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

Practical Simplifications

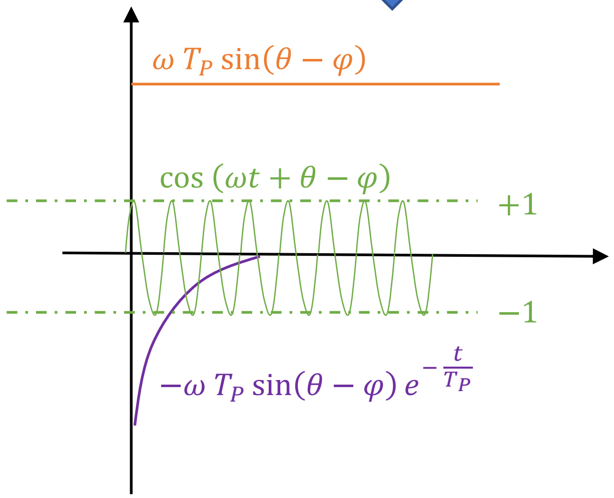
$in\ closed\ core\ CT's\ T_S \gg T_P\ then\ T_S - T_P \approx T_S\ then:$

$$K_S = \frac{\omega T_S\, T_P}{\approx T_S} \sin(\theta-\varphi)\, e^{-\frac{t_S}{T_S}} - \frac{\omega T_S\, T_P}{\approx T_S} \sin(\theta-\varphi)\, e^{-\frac{t_S}{T_P}} + \cos(\omega t_S + \theta - \varphi) + [K_R K_S - \cos(\theta-\varphi)]\, e^{-\frac{t_S}{T_S}}$$

$$K_S = \omega\, T_P \sin(\theta-\varphi)\, e^{-\frac{t_S}{T_S}} - \omega\, T_P \sin(\theta-\varphi)\, e^{-\frac{t_S}{T_P}} + \cos(\omega t_S + \theta - \varphi) + [K_R K_S - \cos(\theta-\varphi)]\, e^{-\frac{t_S}{T_S}}$$

$for\ typical\ values\ of\ T_S\ (e.g.\ 10\ sec),\ during\ first\ few\ cycles\ (below\ 100ms),\ the\ magnetude\ of\ e^{\frac{t_S}{T_S}}\ will\ be\ approximately\ 1:$

$$K_S = \omega\, T_P \sin(\theta-\varphi) \times 1 - \omega\, T_P \sin(\theta-\varphi)\, e^{-\frac{t_S}{T_P}} + \cos(\omega t_S + \theta - \varphi) + [K_R K_S - \cos(\theta-\varphi)] \times 1$$



$for\ the\ wosrt\ case\ scenario\ max\ of\ \cos(\omega t_S + \theta - \varphi)\ should\ be\ 1\ and\ same\ direction\ with\ [\omega\, T_P \sin(\theta-\varphi)]\ component\ (or\ same\ sign\ with\ \sin(\theta-\varphi))$

$$K_S = \omega\, T_P \sin(\theta-\varphi) - \omega\, T_P \sin(\theta-\varphi)\, e^{-\frac{t_S}{T_P}} + 1 + K_R K_S - \cos(\theta-\varphi)$$

$Same\ sign\ with\ \sin(\theta-\varphi)$

$or:$

$$1 - \frac{(1-K_R)\, K_S + \cos(\theta-\varphi) - 1}{\omega\, T_P \sin(\theta-\varphi)} = e^{-\frac{t_S}{T_P}}$$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

Time to saturation

$$1-\frac{(1-K_R)\,K_S+\cos(\theta-\varphi)-1}{\omega\,T_P\sin(\theta-\varphi)}=e^{-\frac{t_S}{T_P}}$$

Finally

$$\boxed{t_S=-T_P\times\ln\left[1-\frac{(1-K_R)\,K_S+\cos(\theta-\varphi)-1}{\omega\,T_P\sin(\theta-\varphi)}\right]}$$

$$\textbf{\textit{substitute}}\ "\omega\,T_P=\frac{X}{R}"$$

Same sign with $\sin(\theta-\varphi)$

$$\boxed{t_S=-\frac{X}{\omega R}\times\ln\left[1-\frac{(1-K_R)\,K_S+\cos(\theta-\varphi)-1}{\frac{X}{R}\sin(\theta-\varphi)}\right]}$$

$$\boxed{K_S=\frac{\left[1-\cos(\theta-\varphi)+\frac{X}{R}\sin(\theta-\varphi)\times\left(1-e^{-\frac{\omega R}{X}t_S}\right)\right]}{(1-K_R)}}$$

$K_S=\dfrac{V_K}{R_b\times I'_{SC}}$ *where* I'_{SC} *is the maximum AC symmetrical short circuit current divided by CT ratio.* V_K *is CT knee point voltage.* $R_b=R_{CT}+R_{EXT}$

K_R: *per unit value of the remanence flux compare to the max flux (knee point flux)*

θ: *voltage angle at the instant of short circuit.* φ: *angle of thevenin impedance at the point of short circuit* $\left(\varphi=\tan^{-1}\dfrac{X}{R}\right)$.

X: *reactance component of thevenin impedance at the point of short circuit.* R: *Resistive component of thevenin impedance at the point of short circuit*

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

Time to saturation

$worst\ time\ of\ short\ circuit\ initiation\ is\ when\ \sin(\theta-\varphi)=1\ or\ \theta-\varphi=90°\ so:$

$$t_S=-\frac{X}{\omega R}\times\ln\left[1-\frac{(1-K_R)\,K_S+\ \cos(\theta-\varphi)-1}{\frac{X}{R}\ \sin(\theta-\varphi)}\right]=-\frac{X}{\omega R}\times\ln\left[1-\frac{(1-K_R)\,K_S+0-1}{\frac{X}{R}\times 1}\right]$$

$$\boxed{t_S=-\frac{X}{\omega R}\times\ln\left[1-\frac{(1-K_R)\,K_S-1}{\frac{X}{R}}\right]}\qquad \boxed{K_S=\frac{\left[1+\frac{X}{R}\times\left(1-e^{-\frac{\omega R}{X}t_S}\right)\right]}{(1-K_R)}}$$

$More\ important\ in\ CT\ sizing\ is\ calculating\ K_S\ when\ t_s\ is\ equal\ to\ relay\ operating\ time\ or\ relay\ pick\ up\ time\ t_{PK}$

$Re\ arrangement\ of\ t_S\ equation\ by\ calculating\ K_S\ when\ t_{PK}=\ t_S\ ,another\ name\ for\ K_S\ in\ this\ case\ is$ "CT $dimensioning\ factor$"

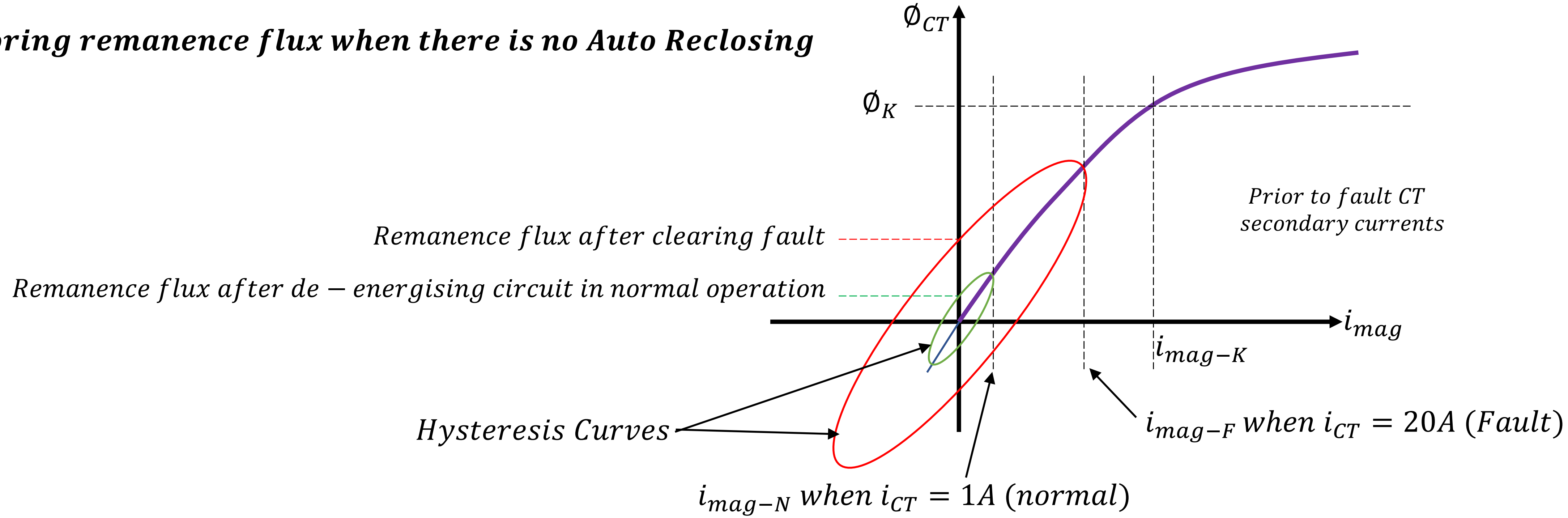
$$t_S=t_{PK}\ \Rightarrow\ \boxed{K_S=\frac{\left[1+\frac{X}{R}\left(1-e^{-\frac{\omega R}{X}t_{PK}}\right)\right]}{(1-K_R)}}$$

$$t_{PK}=\infty\ \Rightarrow\ \boxed{K_S=\frac{\left[1+\frac{X}{R}\right]}{(1-K_R)}}$$

$Conservative\ value\ for\ K_S\ ,if\ K_S\ equal\ to\ this\ value\ then\ CT\ will\ never\ saturate\ for\ the\ highest\ fault$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

Ignoring remanence flux when there is no Auto Reclosing



Refer to the above graph, the magnitude of the remanence flux in normal operation is very low compare to the CT knee point flux. In most applications, after tripping CB, circuit is not reenergised until fault is removed. As such, this is not a practical scenario to switch on fault with high remanence flux $\emptyset_K$ *(from previous fault). The only exception is after re – closing in Auto Reclosig scheme where there is a high chance to switch on to fault with high CT remanence flux left from first operation.*
For this practical reason, we can ignore CT remanace flux for most applications (when there is no auto reclosing).
By applying $K_R = 0$*, equation of CT dimensionig factor* (K_S) *is simplified to:*

$$K_S = \frac{\left[1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X} t_{PK}}\right)\right]}{(1 - K_R)} \xRightarrow{K_R=0} \boxed{K_S = 1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X} t_{PK}}\right)}$$

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

$\boldsymbol{Example\ 1 - \ of\ impact\ of\ relay\ operation\ time\ on\ CT\ dimensionig\ factor}$

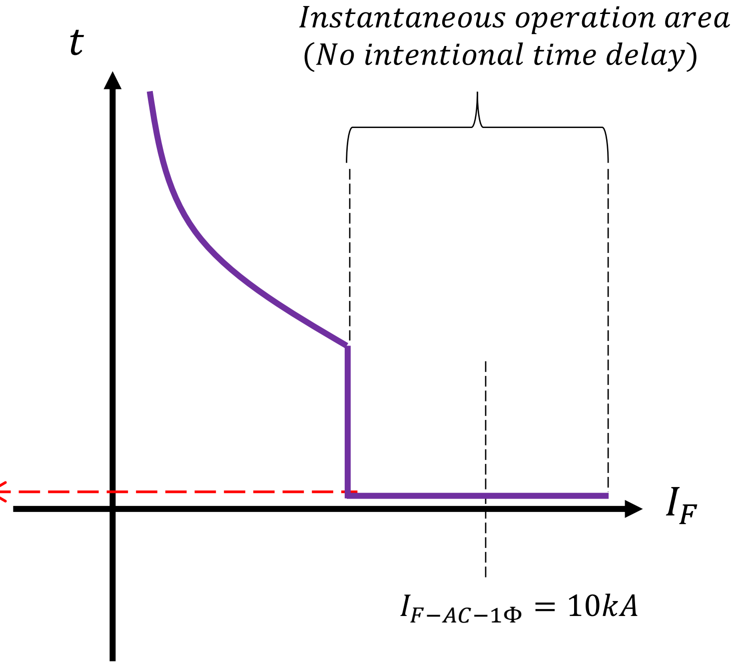
$CT{:}\frac{250}{1}\ , I_{F-AC-1\Phi} = 10kA, \frac{X}{R} = 10, R_b = 4\Omega$

$t_{PK1} = \frac{1}{2}\ Cycle = 10ms = 0.01sec\ Relay\ 1\ Pickup\ (operation)time$

$t_{PK2} = 1.5Cycle = 30ms = 0.03sec\ Relay\ 2\ Pickup\ (operation)time$

$$K_S = 1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_{PK}}\right)$$

t_{PK} : $Relay\ Pickup\ time\ (Operating\ time)$



$$K_{S1} = 1 + 10 \times (1 - e^{-\ 0.1 \times 2\pi \times 50 \times 0.01}) = 3.7 \xrightarrow{Required\ CT\ knee\ point\ voltage\ V_{K1} = K_{S1} \times R_b \times \frac{I_F[A]}{CTR}} V_{K1} = 3.7 \times 4 \times \frac{10000}{250} = 592\ V$$

$$K_{S2} = 1 + 10 \times (1 - e^{-\ 0.1 \times 2\pi \times 50 \times 0.03}) = 7.1 \xrightarrow{Required\ CT\ knee\ point\ voltage\ V_{K2} = K_{S2} \times R_b \times \frac{I_F[A]}{CTR}} V_{K2} = 7.1\ \times 4 \times \frac{10000}{250} = 1136\ V$$

$Min\ required\ Knee\ point\ voltage\ of\ relay\ 1\ CT\ with\ 10ms\ operating\ time\ is\ less\ than\ minimum\ required\ Knee\ point\ voltage\ of\ relay\ 2\ CT\ with\ 30ms\ operating\ voltage.$

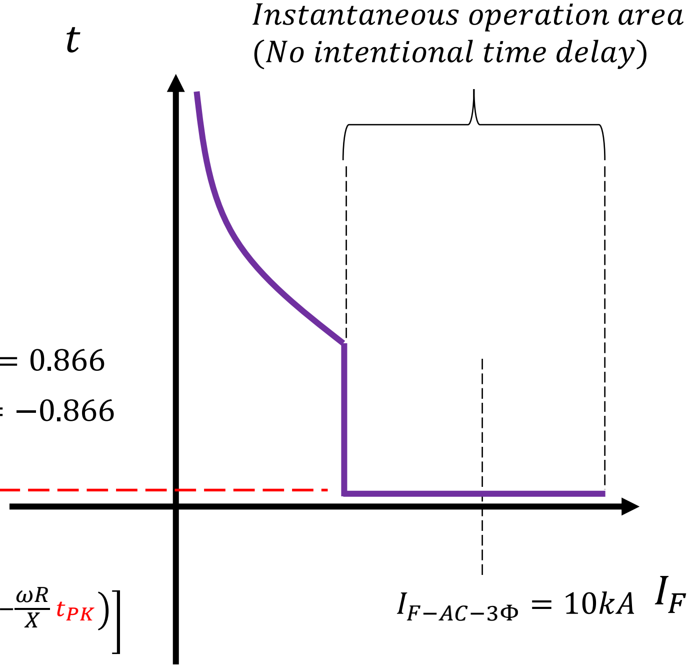
Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

Example 2 – Calculate Min required CT knee point voltage for 3Φ SC

$CT: \frac{250}{1}, I_{F-AC-3\Phi} = 10kA, \frac{X}{R} = 10, R_b = 4\Omega$

$t_{PK} = 1.5\ Cycle = 30ms = 0.03sec\ Relay\ Pickup\ (operation) time$

$$Worst\ case\ switching\ instant\ at\ \phi_A \begin{cases} \theta_{\phi A} - \varphi = 90°\ So: \sin(\theta_{\phi A} - \varphi) = 1\ and\ \cos(\theta_{\phi A} - \varphi) = 0 \\ \theta_{\phi B} - \varphi = 240° + \theta_{\phi A} - \varphi = 240° + 90° = 330°\ So: \sin(\theta_{\phi B} - \varphi) = -0.5\ and\ \cos(\theta_{\phi B} - \varphi) = 0.866 \\ \theta_{\phi C} - \varphi = 120° + \theta_{\phi A} - \varphi = 120° + 90° = 210°\ So: \sin(\theta_{\phi C} - \varphi) = -0.5\ and\ \cos(\theta_{\phi C} - \varphi) = -0.866 \end{cases}$$



Same sign with $\sin(\theta - \varphi)$

$$K_S = \frac{\left[1 - \cos(\theta - \varphi) + \frac{X}{R}\sin(\theta - \varphi) \times \left(1 - e^{-\frac{\omega R}{X} t_{PK}}\right)\right]}{(1 - K_R)} \xrightarrow{K_R = 0\ (ignore\ remanence\ flux)} K_S = \left[1 - \cos(\theta - \varphi) + \frac{X}{R}\sin(\theta - \varphi) \times \left(1 - e^{-\frac{\omega R}{X} t_{PK}}\right)\right]$$

$$\begin{cases} \phi_A: K_{SA} = [1 - 0 + 10 \times 1 \times (1 - e^{-2\pi 50 \times 0.1 \times 0.03})] = 7.1 \xrightarrow{Required\ CT\ knee\ point\ voltage\ V_{K-A} = K_{SA} \times R_b \times \frac{I_F[A]}{CTR}} V_{k-A} = 7.1 \times 4 \times \frac{10000}{250} = 1136\ V \\ \phi_B: K_{SB} = \left|[-1 - 0.866 + 10 \times (-0.5) \times (1 - e^{-2\pi 50 \times 0.1 \times 0.03})]\right| = |-4.92| = 4.92 = 0.7K_{SA} \xrightarrow{Required\ CT\ knee\ point\ voltage\ V_{K-B} = K_{SB} \times R_b \times \frac{I_F[A]}{CTR}} V_{k-B} = 4.92 \times 4 \times \frac{10000}{250} = 787.2\ V \\ \phi_C: K_{SC} = \left|[-1 - (-0.866) + 10 \times (-0.5) \times (1 - e^{-2\pi 50 \times 0.1 \times 0.03})]\right| = |-3.2| = 3.2 = 0.45K_{SA} \xrightarrow{Required\ CT\ knee\ point\ voltage\ V_{K-C} = K_{SC} \times R_b \times \frac{I_F[A]}{CTR}} V_{k-C} = 3.2 \times 4 \times \frac{10000}{250} = 512\ V \end{cases}$$

All three phases are not in their worst case switching instant in 3ϕ short circuits (instantaneous protections). This example shows that when ϕ_A is at worst case switching with calculated CT over dimensiong factor of K_{SA}, at ϕ_C, $K_{SC} = 0.45K_{SA}$.
As such, in 3ϕ SC, due to smaller SC DC component in other phases, CT over dimensiong factor of worst case switching should be reduced by approximately 50% (≈ multipliying by $|\sin(\theta - \varphi)|$ of other phases). In this example, min $V_K = 512V$.

Protection Current Transformers (CT's) – Time to Saturation AND CT Over Dimensioning Factor

Simplification of K_S *equation* :

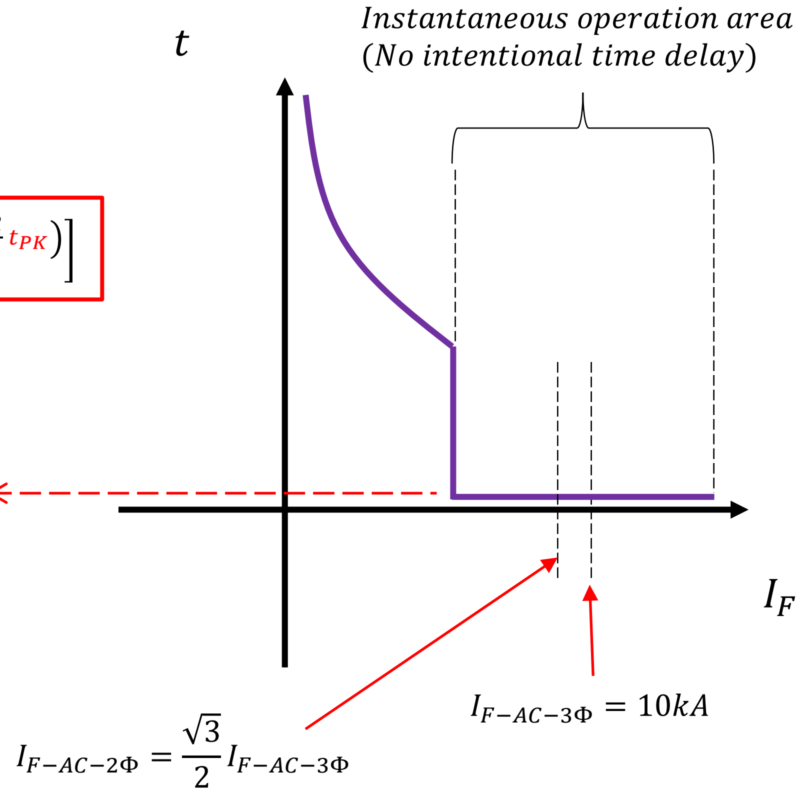
Same sign with $\sin(\theta-\varphi)$

$$K_S=\frac{\left[1-\cos(\theta-\varphi)+\frac{X}{R}\sin(\theta-\varphi)\times\left(1-e^{-\frac{\omega R}{X}t_{PK}}\right)\right]}{(1-K_R)}\xrightarrow{\text{In practical scenarios}}\approx\boxed{K_S=\frac{|\sin(\theta-\varphi)|}{(1-K_R)}\times\left[1+\frac{X}{R}\times\left(1-e^{-\frac{\omega R}{X}t_{PK}}\right)\right]}$$

Example 3 – Calculate Min required CT knee point voltage for 2Φ SC

$t_{PK}=1.5\ Cycle=30ms=0.03sec\ Relay\ \ Pickup\ (operation)time$

t_{PK} : *Relay Pickup time (Operating time)*



$$CT:\frac{250}{1},I_{F-AC-3\Phi}=10kA,I_{F-AC-2\Phi}=\frac{\sqrt{3}}{2}10kA=8.67\ kA,\frac{X}{R}=10,R_b=4\Omega$$

$t_{PK}=1.5\ Cycle=30ms=0.03sec\ Relay\ \ Pickup\ (operation)time$

$$K_S=\frac{|\sin(\theta-\varphi)|}{(1-K_R)}\times\left[1+\frac{X}{R}\times\left(1-e^{-\frac{\omega R}{X}t_{PK}}\right)\right]\xrightarrow{\sin(\theta-\varphi)=0\ \&K_R=0}K_S=\frac{1}{1}\times\left[1+10\times\left(1-e^{-0.1\times2\pi\times50\times0.03}\right)\right]=7.1\xrightarrow{\text{Required CT knee point voltage } V_K=K_S\times R_b\times\frac{\frac{\sqrt{3}}{2}I_{F-3\emptyset}[A]}{CTR}}$$

$$V_K=7.1\times4\times\frac{\frac{\sqrt{3}}{2}\times10000}{250}=984\ V$$

Protection Current Transformers (CT’s) – Time to Saturation AND CT Over Dimensioning Factor

Conclusions:

In the ideal situation, CT's shall not be saturated for any through faults to give the exact image of the primary fault current to the protection relays.

Due to high asymmetrical fault component (DC component) of SC in HV power systems, CT's can be saturated at the initiation of the faults if their knee point voltages only enough to handle AC symmetrical fault current. As such when calculating required CT knee point voltage (V_K), *CT over dimensioning factor* (K_S) *is calculated as well and is multiplied to the short circuit AC symmetrical component to make sure that CT is not saturated during short circuit. However, it is not always practical to have CT with required* V_K, *e.g. assume a 100 to 1 CT in a system with 10kA SC, the secondary current will be 100A during SC and saturation is almost inevitable in the standard CT' in this range.*

One of methods for finding required CT knee point voltage in instantaneous protections (such as distance zone 1, instantaneous overcurrent) is to calculate CT time to saturation then calculating required CT knee point voltage in a way that protection relays measure short circuit current accurately prior to CT saturation

This presentation demonstrates equations and circuit theory bakground of finding CT over dimensioning factor based on CT time to saturation.

There are several researches and patents (from relay manufacturers)in this topic about methods of current measuring in different protection relays (e.g. some of them can detect CT saturation)which are beyond this presentation. In practical cases, protection relays' OEM (Original Equipment Manufacturer) recommendations shall be followed.

Appendix A – Details of calculation of $i_S(t)$

$$\frac{di_S(t)}{dt} + \frac{1}{T_S}\, i_S\,(t) = -\frac{I'_m}{T_P}\sin(\theta-\varphi)\; e^{-\frac{t}{T_P}} + I'_m\,\omega\; \cos(\omega t+\theta-\varphi)$$

$\Downarrow$

$$i_S(t) = i_{S0}(t) + i_{S1}(t) + i_{S2}(t) = K_0\, e^{-\frac{t}{T_S}} + K_1\, e^{-\frac{t}{T_P}} + K_2\; \sin(\omega t+\beta)$$

$\Downarrow$

$$i_{S1}(t) = K_1\, e^{-\frac{t}{T_P}} = Particular\ Solution\ for\ i_S(t)\ when\ diffrential\ equation = -\frac{I'_m}{T_P}\sin(\theta-\varphi)\; e^{-\frac{t}{T_P}}\ (needs\ to\ calculate\ K_1)$$

$$i_{S2}(t) = K_2\; \sin(\omega t+\beta) = Particular\ Solution\ for\ i_S(t)\ when\ diffrential\ equation = I'_m\,\omega\; \cos(\omega t+\theta-\varphi)\,(needs\ to\ calculate\ K_2\ \&\ \beta)$$

$$i_{S0}(t) = K_0\, e^{-\frac{t}{T_S}} = Natural\ \ Responce\ of\ diffrential\ equation\ (needs\ to\ calculate\ K_0\ after\ calculating\ i_{S1}(t)\ \&\ i_{S2}(t)\ then\ applying\ i_S(0)=0)$$

$\boldsymbol{Appendix\ A\ -Details\ of\ calculation\ of\ i_S(t)}$

$\boldsymbol{Calculating\ i_{S1}(t)}$

$$-\frac{K_1}{T_P}\,e^{-\frac{t}{T_P}}+\frac{K_1}{T_S}\,e^{-\frac{t}{T_P}}=-\frac{I'_m}{T_P}\sin(\theta-\varphi)\ e^{-\frac{t}{T_P}}\ \rightarrow\rightarrow\rightarrow\quad K_1=-\frac{T_S}{T_S-T_P}\,I'_m\sin(\theta-\varphi)$$

$$\boxed{i_{S1}(t)=-\frac{T_S}{T_S-T_P}\,I'_m\sin(\theta-\varphi)\,e^{-\frac{t}{T_P}}}$$

$\boldsymbol{Calculating\ i_{S2}(t) by\ time\ domain\ method}$

$$\frac{di_{S2}(t)}{dt}+\frac{1}{T_S}\,i_{S2}\,(t)=I'_m\,\omega\,\cos(\omega t+\theta-\varphi)\ AND\ i_{S2}(t)=K_2\,\sin(\omega t+\beta)$$

$$K_2\,\omega\cos(\omega t+\beta)+\frac{1}{T_S}\,K_2\,\sin(\omega t+\beta)=I'_m\,\omega\,\cos(\omega t+\theta-\varphi)\rightarrow\ K_2\left[\cos(\omega t+\beta)+\frac{1}{\omega T_S}\sin(\omega t+\beta)\right]=I'_m\,\cos(\omega t+\theta-\varphi)$$

$$Assume\ \tan\varphi_{CT}=\frac{1}{\omega T_S}=\frac{R_b}{\omega L_m}=\frac{R_b}{X_m}=1\Big/\frac{X_m}{R_b}$$

$Appendix\ A\ -\ Details\ of\ calculation\ of\ i_S(t)$

$Calculating\ i_{S2}(t) by\ time\ domain\ method\ -\ Continue$

$$K_2\,[\cos(\omega t+\beta)\ +\ \tan\varphi_{CT}\ \sin(\omega t+\beta)] = I'_m\cos(\omega t+\theta-\varphi)\ \rightarrow K_2\,[\cos(\omega t+\beta)\ +\ \frac{\sin\varphi_{CT}}{\cos\varphi_{CT}}\ \sin(\omega t+\beta)] = I'_m\cos(\omega t+\theta-\varphi)$$

$$\cos(x-y) =\ \cos x\cos y +\ \sin x\ \sin y\,,\qquad \text{assume } x = \omega t+\beta\ and\ y =\ \varphi_{CT}$$

$$\frac{K_2}{\cos\varphi_{CT}}\cos(\omega t+\beta-\varphi_{CT}) = I'_m\cos(\omega t+\theta-\varphi)$$

$$\beta = \theta-\varphi+\varphi_{CT}\ since\ X_m \gg R_b\ then\ \tan\varphi_{CT} = \frac{R_b}{X_m}\approx 0 \text{ then } \varphi_{CT}\approx 0\ then \qquad \boxed{\beta\approx\theta-\varphi}$$

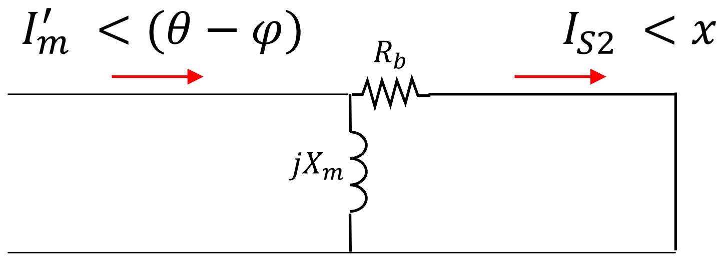
$$sin^2x +\ cos^2x = 1\ \rightarrow\ tan^2x+1 = \frac{1}{cos^2x}\ \rightarrow\ \cos x = \frac{1}{\sqrt{1+\ tan^2x}}$$

$$\frac{K_2}{\cos\varphi_{CT}} = K_2\sqrt{1+tan^2_{\varphi CT}} = K_2\sqrt{1+\frac{R_b^2}{X_m^2}} = \frac{K_2\sqrt{R_b^2\ +X_m^2}}{X_m} = I'_m \rightarrow K_2 = \frac{X_m}{\sqrt{R_b^2+X_m^2}}\,I'_m\ \rightarrow X_m \gg\ R_b\ then \qquad \boxed{K_2\approx I'_m}$$

$$\boxed{i_{S2}(t) = K_2\ \sin(\omega t+\beta) = I'_m\sin(\omega t+\theta-\varphi)}$$

$\boldsymbol{Appendix\ A\ - Details\ of\ calculation\ of\ i_S(t)}$

$\boldsymbol{Calculating\ i_{S2}(t) by\ phasor\ (\ frequency\ method)}$



$i_{S2}(t)$ *is solution for a sine wave and it can be calculated by phasor*

$$I'_m \sin(\omega t + \theta - \varphi) \equiv I'_m < (\theta - \varphi)$$

$$I_{S2} < x = I'_m < (\theta - \varphi)\frac{jX_m}{R_b + jX_m} = I'_m < (\theta - \varphi)\frac{X_m < 90°}{|R_b + jX_m| < \tan^{-1}(\frac{X_m}{R_b})} = \frac{I'_m X_m < (\theta - \varphi + 90° - \tan^{-1}(\frac{X_m}{R_b}))}{\sqrt{R_b^2 + X_m^2}}$$

$$X_m \gg R_b\ then: \frac{X_m}{\sqrt{R_b^2 + X_m^2}} \approx 1\ AND\ \tan^{-1}\left(\frac{X_m}{R_b}\right) \approx 90°\ So:\ I_{S2} < x = \frac{I'_m X_m < (\theta - \varphi + 90° - \tan^{-1}(\frac{X_m}{R_b}))}{\sqrt{R_b^2 + X_m^2}} \approx I'_m < (\theta - \varphi)$$

$$\boxed{So:\ I_{S2} \approx I'_m\ and\ x \approx (\theta - \varphi)}$$

$$\boxed{i_{S2}(t) = I'_m \sin(\omega t + \theta - \varphi)}$$

Appendix A − Details of calculation of $i_S(t)$

Calculating $i_{S0}(t)$ and $i_S(t)$

$$i_S(t) = i_{S0}(t) + i_{S1}(t) + i_{S2}(t) = K_0\, e^{-\frac{t}{T_S}} + K_1\, e^{-\frac{t}{T_P}} + K_2\, \sin(\omega t + \beta)$$

$$i_S(t) = K_0\, e^{-\frac{t}{T_S}} - \frac{T_S}{T_S - T_P}\, I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}} + I'_m\, \sin(\omega t + \theta - \varphi)$$

$$i_S(t = 0) = 0$$

$$0 = K_0\; - \frac{T_S}{T_S - T_P}\, I'_m \sin(\theta - \varphi) + I'_m\, \sin(\theta - \varphi)$$

$$K_0 = \frac{T_S}{T_S - T_P}\, I'_m \sin(\theta - \varphi) - I'_m\, \sin(\theta - \varphi) = \frac{T_P}{T_S - T_P} I'_m \sin(\theta - \varphi)$$

$$\boxed{i_S(t) = \frac{T_P}{T_S - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_S}} - \frac{T_S}{T_S - T_P}\, I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}} + I'_m\, \sin(\omega t + \theta - \varphi)}$$